OFFICE OF THE SUPERINTENDENT OF SCHOOLS
Peterborough, New Hampshire

CONTOOCOOK VALLEY SCHOOL BOARD

Strategic Plan Committee
SAU Office/Boardroom

Thursday, April 13, 2017
7:00 p.m.

MINUTES

School Board Committee Members:
- Pierce Rigrod
- Richard Dunning
- Bernd Foecking
- Jim Fredrickson
- Kristen Reilly

Committee Members Present: Pierce Rigrod, Richard Dunning, Jim Fredrickson, Kristen Reilly

Others Present: Riley Young, John Jordan (SAC), Bob Edwards, Stephan Morrissey, Kimberly Saunders (7:20), Ann Forrest (7:22), Rich Cahoon (7:34), Myron Steere (7:34)

Pierce Rigrod called the meeting to order at 7:02 p.m.

1. Approval of Minutes from March 9, 2017
Dick Dunning moved to accept the minutes of March 9, 2017. Pierce Rigrod second. Kristen abstained.

2. Election of Committee Chair
Dick Dunning recommended that Pierce Rigrod continue as Chair. Second. Unanimous.

3. Strategic Plan – Implementation (SAU/Board)
Pierce Rigrod spoke about Summer Program and the progress being made. The numbers of students taking summer program to catch up to extending learning has grown. Progress is underway in terms of discussion on the high school renovation.
Dick Dunning reported out on a meeting with Hutter Construction. The agenda is to go back to the high school to discuss the vision for the labs and the renovations to the core structure. The expenditure will be looked at. The hope is to use revenue that we have to attack that and not have to bond. The goal is to have this in line for a decision prior to July 1st. Realistic figures need development so decisions can be made. The first priority is renovation of the science labs.
The district started the OGAP program to strengthen math. It was a big investment. SWIFT is also in process; it involves inclusion. In addition, the Education Equity Report is underway by Ann Forrest. The technology plan is moving forward.
The Strategic Plan is a roadmap that we are on.

4. Configuration Models – Model Weights & Companion Reports (SAU)
Four models; things that we could different that would be better in a number of ways.
Reconfiguration – K-8, soft borders (go to school closest to where you live), close schools (reducing the number of buildings), and status quo.
Status Quo – based on birth rates, what will the district look like five years from now?
Reconfiguration – use the buildings we have now; how could it look different? Potential Pre-K/1 centers. What would it cost? Could it save us?

Consolidation – closing a school or schools. Could we offer foreign languages? Could we offer 1:1 experiences? Universal Preschool? What could we do that we can't now if we consolidated?
Nothing is off the table until it doesn't make sense for who we are.
Administration will make a recommendation with each report.
The expectation is that information for Board review should be ready by the end of June.
Ultimately the board will make the decision on what they want to move forward.

Weights and Measures – Deciding how it should be weighted and what will be weighted needs determination ahead of time.
A normative process might be considered in terms of what is important to people; what do they value?
Trend data might be a place to start.
Jim Fredrickson will work on criteria for the next meeting. A sample matrix will come forward.
Each board sub-committee should be given deadlines for information to get the work done.

5. Facilitation of Models Discussion (timeline)
Companion reports will come in June. The board will decide which models to pursue in public. Facilitation would follow to work through the hard parts and arrive at consensus. Information will be shared with the public.
The board needs to identify priorities i.e. shorten transportation time for students.
Kimberly Saunders will facilitate the discussion that will inform the work. Jim Fredrickson and Kimberly will share at the May retreat. From there, options go into the matrix, then out to the community for sharing.
Gaining community input on what is important.
Do not inject public process to affect criteria. Develop criteria internally. Confirmed.
Important factors – education, transportation, facility cost, and staffing cost.

6. Financial Equity Study (distribution & comment through May)
The Financial Equity Study report has been distributed minimally.
An Executive Summary was suggested along with tables for public consumption.
Baseline information using State calculations and other information was suggested. What would it look like for each town for comparison?
A Peer Review Letter should be included in the report.
Town by town impact should be considered.

7. Policies Related to Strategic Planning
(Multi-Age, attending school other than assigned school (status))
Multi-age and Multi-grade is still with the Education Committee. What are the educational opportunities?
Policy JCA is used on a regular basis. There is no solid reason to change the policy to attend a school other than assigned.
Are there other policies that need review? Not at this point.

8. Other
Kristen Reilly will serve as the Strategic Plan Committee rep to the Communications Committee.

Dick Dunning motioned to adjourn at 8:15 p.m. Second. Unanimous.

Respectfully submitted,

Brenda Marschok

Strategic Plan Updates:

Goal 1

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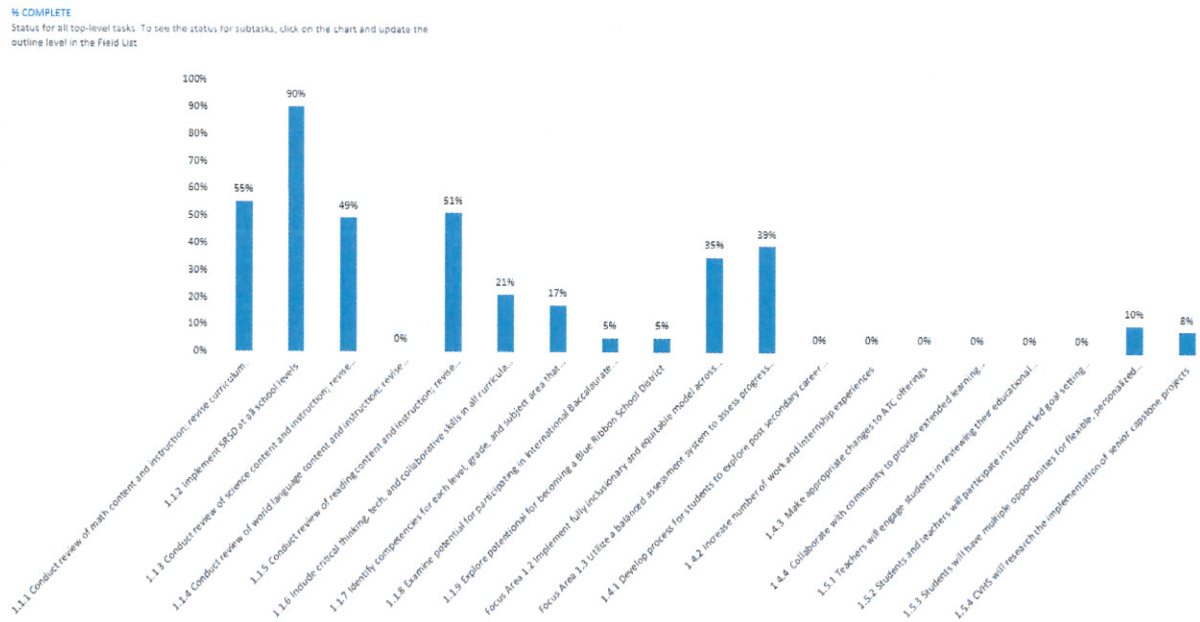

Goal 2 Project Overview

TUE 9/1/15 - WED 6/30/21

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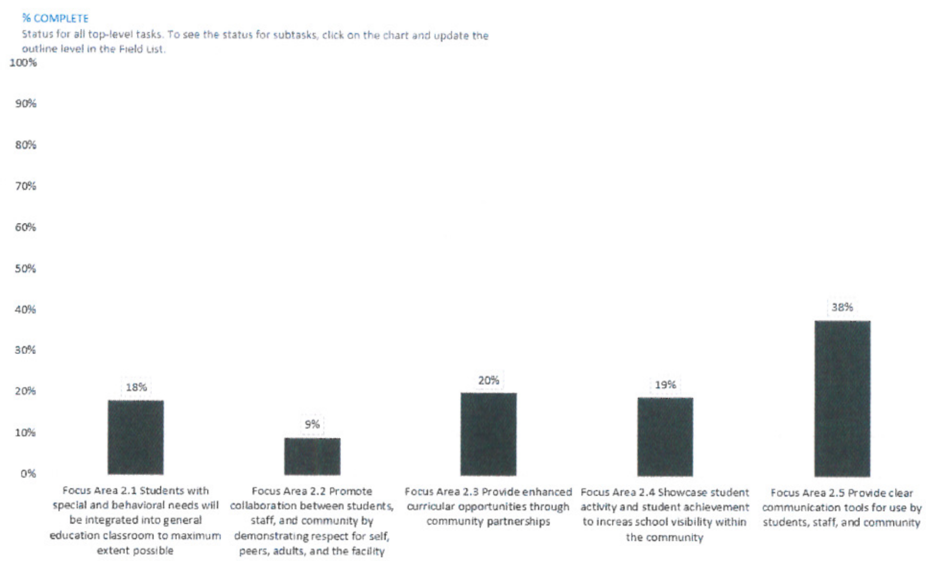

Goal 3 PROJECT OVERVIEW

MON 6/1/15 - WED 6/30/21

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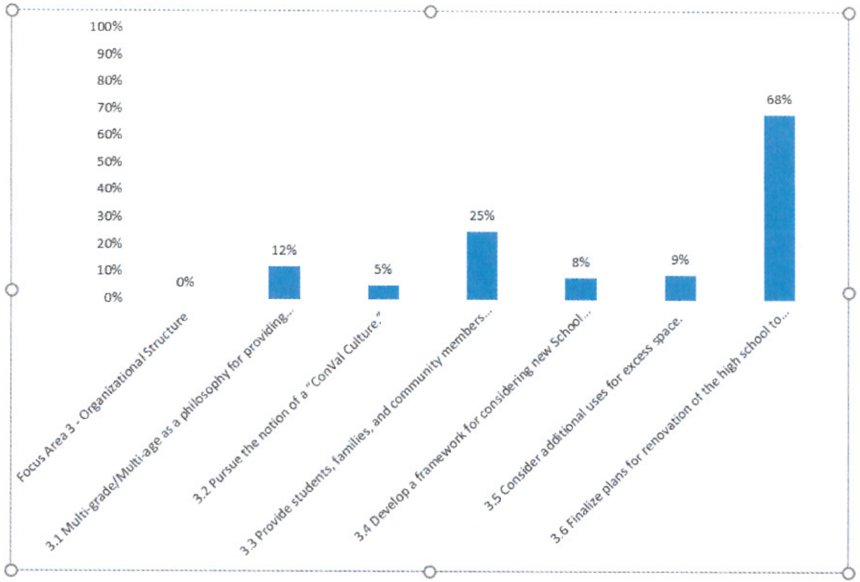
Status for all top-level tasks. To see the status for subtasks, click on the chart and update the outline level in the Field List.



Goal 4 Project Overview

WED 8/5/15 - FRI 6/3/22

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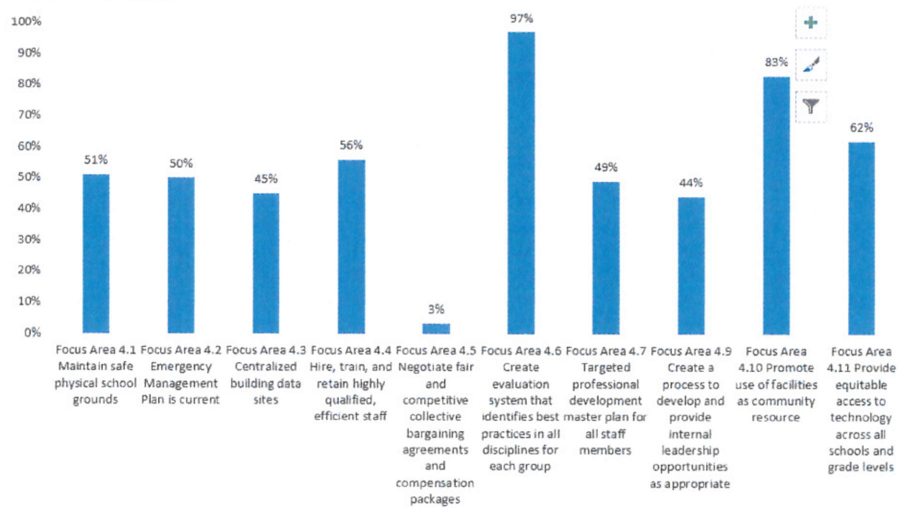
Status for all top-level tasks. To see the status for subtasks, click on the chart and update the outline level in the Field List.



Bond:

The initial Bond report was submitted on 11/21/2017. Since that time there have been several considerations, including prioritizing and pricing out the project, paring the project down significantly, as well as looking at performing the work only on the Science Labs. Thoughts have included trying to bring the Bond to a more palatable cost point as well as trying to see if we can use Capital Reserve dollars. On 4/12/2017 Dick Dunning, Myron Steere, and Tim Grossi met with Hutter regarding the project to see what insight could be offered and what cost savings could be realized.

Balanced Assessment/Benchmark for Student Assessment/Achievement

These items are presently in discussion at the Ed Committee level. A model to describe Balanced Assessment has been presented to Education Committee and an application for PACE is also in consideration as part of a Balanced Assessment System. Presently due to changes at the DOE the status of PACE is uncertain. The topic of a growth year has been under discussion by the administration. We are presently considering a combination of NWEA growth target and AIM's Web (K-4) growth rate to help us identify a metric for a year's growth for each individual student. This will be discussed at the Education Committee once a final recommendation is ready, which will be by the end of May at the latest.

ATC programming

The following is an update to me from John Reitnaauer, based on this information he is presently examining the costs and needs associated to this potential programming:

March update ATC

Employment opportunities in the areas of health care and construction will see significant increases by the year 2022. According to the New Hampshire Employment Projections by Industry and Occupation: Base year 2012 to projected year 2022 report (Hassan, Copadis, & Demay, 2014), the health care and social assistance sector is expected to add 30% of all new jobs. Employment in construction is expected to grow by 17.7%. Within the construction sector, specialty trade contractor employment will increase by 19%.

Based on these data, the Regional Advisory Committee should explore the possibility of adding the following Career and Technical Education Programs over the next five years: Health Professions and Related Services (program code 519999), Electrician (program code 460302), and Plumbing and Water Supply Services (program code 460599). Current programs offered at Con Val High School and the Region 14 ATC would compliment these program additions. The Anatomy and Physiology 1 and 2, LNA, and EMT programs would be included in the Health Professions program. The Electrician and Plumbing and Water Supply Services programs would compliment the current Building/Construction Trades program.

References

Hassan, M.W., Copadis, G.N., DeMay, B.R. (2014, June). New Hampshire Employment Projections by Industry and Occupation: Base year 2012 to projected year 2022. New Hampshire Employment Security Economic & Labor Market Information Bureau. Retrieved from www.nh.gov/nhes/elmi/projections.htm

Summer Programming:

There are several summer programming opportunities being planned. At the two middle schools there will be summer programming in each building. Presently the Middle School principals are working in conjunction with Dr. Forrest to decide what will be offered, when it will be offered, and who is most able to provide specific programming.

Respectfully submitted

Kimberly Rizzo Saunders

Weighted Criteria Matrix

Description

The weighted criteria matrix is a valuable decision-making tool that is used to evaluate program alternatives based on specific evaluation criteria weighted by importance. By evaluating alternatives based on their performance with respect to individual criteria, a value for the alternative can be identified. The values for each alternative can then be compared to create a rank order of their performance related to the criteria as a whole. The tool is important because it treats the criteria independently, helping avoid the over-influence or emphasis on specific individual criteria.

The matrix itself is constructed with the alternatives listed along one side and the review criteria along the other. A box to insert the specific assigned weight is located with each criteria. An evaluation scale is established for the whole matrix. The ranking of the alternative based on its ability to address the specific criteria is entered into the appropriate cell. The total scores are then available to use in ranking alternatives.

Why Use Weighted Criteria Matrix to Develop and Review Workplace Solutions?

- The weighted criteria matrix is just one means of evaluating proposed workplace strategies. Often, rather than not having enough ideas for the strategies, organizations find themselves with too many. The weighted criteria matrix can help organizations narrow the list of options using criteria such as cost against other criteria such as quality or efficiency.

How Else Can It Be Used?

- Use the weighted criteria matrix whenever a decision or series of decisions are necessary in an IWS project.

- **Managing Change:** It can also be used to help resolve conflicts during the change management stage of an IWS project, and in the establishment of initial project goals.

Who to Involve

- All project team members should be involved in the development of a weighted criteria matrix.

- When decisions will affect user groups, representatives of those groups should have input to the weighted criteria matrix process.

Source: Adapted from Joe Ouye, Facility Technics Facility Management Consulting, 505 17th Street, Suite 300, Oakland, CA 94612. Adapted by permission.

Sample: Weighted Criteria Matrix

	Criterion / *Weight*	Criterion 1 Cost effectiveness for Div 4		Criterion 2 Cost effectiveness for Co 4		Criterion 3 Employee satisfaction 1		Criterion 4 Flexibility for Divs 3		TOTAL SCORE
	ALTERNATIVE	Raw Score	Wtd Score	Raw Score	Wtd Score	Raw Score	Wtd Score	Raw Score	Wtd Score	
1	Status Quo (incl same cost allocation scheme)	0		0		4	4	0		4
2	Keep all Divisions in the same hub office, with private offices	1	4	1	4	4	4	1	3	15
3	Keep all Divisions in the same hub office, with office-sharing (2 salespeople to 1 office)	2	8	2	8	3	3	1	3	22
4	Keep all Divisions in the same hub office, with hoteling (3 salespeople to 1 office)	3	12	3	12	2	2	3	9	35
5	All Divisions to work out of Executive Suites	2	8	2	8	3	3	4	12	31
										0

Cost effectiveness for Div	Cost effectiveness for Co	Employee satisfaction	Flexibility for Divs
0 Low 2 Medium 4 High	0 Low 2 Medium 4 High	0 Low 2 Medium 4 High	0 Low 2 Medium 4 High

Source: Joe Ouye, Facility Technics Facility Management Consulting, 505 17th Street, Suite 300, Oakland, CA 94612. Used by permission.

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Weight determination for consistently ranking alternatives in multiple criteria decision analysis

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TOPSIS

ABSTRACT

One of the most difficult tasks in multiple criteria decision analysis (MCDA) is determining the weights of individual criteria so that all alternatives can be compared based on the aggregate performance of all criteria. This problem can be transformed into the compromise programming of seeking alternatives with a shorter distance to the ideal or a longer distance to the anti-ideal despite the rankings based on the two distance measures possibly not being the same. In order to obtain consistent rankings, this paper proposes a measure of relative distance, which involves the calculation of the relative position of an alternative between the anti-ideal and the ideal for ranking. In this case, minimizing the distance to the ideal is equivalent to maximizing the distance to the anti-ideal, so the rankings obtained from the two criteria are the same. An example is used to discuss the advantages and disadvantages of the proposed method, and the results are compared with those obtained from the TOPSIS method.

1. Introduction

In practice, there are many situations for which the aggregate performance of a group of alternatives must be evaluated based on a set of criteria. The alternative with the best aggregate performance is chosen for implementation. The associated research falls into the category of multiple criteria decision analysis (MCDA).

Numerous MCDA methods for ranking alternatives have been developed [1–3]. The essence of each method is the way that the performances of the selected criteria are aggregated. The study of Eckenrode [4] is a classic work on this topic. Later developments include the partial utility function [5], the analytic hierarchy process [6,7], ordinal regression [8], and the centroid method [9]. Once the importance of each criterion is decided, the aggregate scores are calculated and the rankings are determined.

Following this procedure, the most critical step is determining the importance of each criterion. Fundamentally, there are two ways of eliciting the weights of criterion importance: direct explication and indirect explication [10]. Direct explication refers to eliciting weights through expert interviews, questionnaire surveys, and conventional rules, where weights are determined before the data of each alternative is collected. They are called *a priori* weights. Since the weights show the emphases of the decision maker, they serve as a guide for future development. For example, if the criterion *pollution* has a larger weight than that of *profit*, then more effort will be devoted to pollution control than to profit generation to obtain a higher rank. Indirect explication refers to obtaining weights from the data. Since the weights must be determined after the data of each alternative is collected, they are called *a posteriori* weights. As opposed to direct explication where the weights are the emphases of experts, the weights of indirect explication represent the emphases of the alternatives being evaluated. This way of determining weights is more convincing because the weights are a reflection of the data.

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Another key factor in MCDA is the determination of a benchmark for comparison. Usually, an ideal alternative is identified, and alternatives that are closer to the ideal are preferred. Some studies [10–12] have discussed the idea that being farther away from the negative ideal, or anti-ideal, is better, where the negative ideal is the imaginary alternative which has the smallest value in each criterion. The alternatives are ranked based on their distance to the ideal or anti-ideal. Various ways for measuring the distance have been defined. Let $\boldsymbol{X} = [X_1, X_2, \ldots, X_m]$ and $\boldsymbol{Y} = [Y_1, Y_2, \ldots, Y_m]$ denote two points in m-dimensional space. The generalized distance measure between $\boldsymbol{X}$ and $\boldsymbol{Y}$ is [13]:

$$d_p = \left[\sum_{i=1}^{m} w_i^p (X_i - Y_i)^p\right]^{1/p} \tag{1}$$

where p represents the distance parameter and w_i is the relative importance, represented in weight, of the ith criterion. Usually, $p = 2$ is used.

For two alternatives with the same distance to the ideal, the one which is farther away from the anti-ideal is considered better because it is "relatively" closer to the ideal. Similarly, for two alternatives with similar distances to the anti-ideal, the one which is closer to the ideal is preferred. In this regard, a measure of relative distance which shows the relative position of an alternative from the anti-ideal to the ideal is desirable. This paper formulates the problem of weight determination using a compromise programming technique, where the difference between the performances of the alternative and the ideal is treated as the distance. The rankings of the alternatives are based on the aggregate performance calculated from the set of weights. One attractive feature of the relative distance measure is that the rankings obtained based on the distance to the ideal and those obtained based on the distance to the anti-ideal are the same.

The rest of this paper is organized as follows. The data envelopment analysis (DEA) technique for identifying nondominated alternatives is reviewed in Section 2. Then, the traditional idea of calculating the shortest distance between the observed and the ideal alternatives, yet based on *a posteriori* weights, is given in Section 3. In Section 4, the idea of calculating the relative distance for ranking is proposed. An example is used in Section 5 to compare the proposed method with the TOPSIS method. Finally, in Section 6, some conclusions are drawn.

2. Nondominated alternatives

When more than one criterion is considered, there will usually be several alternatives which are not dominated by the others; each has at least one criterion which outperforms those of the other alternatives. One of the nondominated alternatives is chosen for implementation. Charnes et al. [14] proposed the DEA technique to calculate the relative efficiency of a group of decision making units (DMUs) which uses multiple inputs to produce multiple outputs. Each unit is allowed to use different sets of weights to calculate the efficiency. Those with an efficiency value of 1 are nondominated units, which are called Pareto optimal or efficient units [14]. The MCDA problem can be considered as a DEA problem without inputs, or as a problem in which every alternative has the same amount of every input. Hence, the DEA technique can be applied to identify nondominated alternatives.

Let Y_{ij} denote the value of the ith criterion, $i = 1, \ldots, m$, for the jth alternative, $j = 1, \ldots, n$. The DEA model without inputs for calculating the efficiency of the kth alternative can be formulated as [15]:

$$\begin{aligned} E_k = \max.\ & \sum_{i=1}^{m} w_i Y_{ik} \\ \text{s.t.}\quad & \sum_{i=1}^{m} w_i Y_{ij} \leqslant 1, \quad j = 1, \ldots, n \\ & w_i \geqslant \varepsilon, \quad i = 1, \ldots, m, \end{aligned} \tag{2}$$

where w_i is the importance associated with the ith criterion and ε is a small positive quantity imposed to restrict any criterion from being ignored. The most favorable weights are sought for each DMU in calculating its efficiency. This model has a dual which is exactly the same as the output-oriented BCC model [16] without inputs formulated by Lovell and Pastor [17], who also proved that the dual model is equivalent to the output-oriented CCR model [14] with a single constant input.

Consider a simple example of five alternatives, *A*, *B*, *C*, *D*, and *E*. Their performances in two criteria, Y_1 and Y_2, are shown in Table 1 and are depicted in Fig. 1. Model (2) identifies *B* and *D* as the nondominated alternatives. The piecewise line segments *SBDT* present the efficiency frontier constructed from the five alternatives. Alternative *E* lies on the vertical line extended downward from the efficient alternative *D*, and is called weakly efficient [18]. Alternatives *A* and *C* lie in the interior of the area delineated by line segments *SBDT* and are thus dominated. Their efficiency values, as calculated from Model (2), are the ratios of *OA* to *OA′* and *OC* to *OC′*, respectively, where *A′* and *C′* are the projections on their respective frontier facets. Column 4 of Table 1, with the heading "DEA efficiency", shows the efficiency values of the five alternatives. Numbers in parentheses are their ranks.

In this case, *B* and *D* are the best choices because they are nondominated. However, it is not clear which one is better. Alternative *E* is ranked the next best since it is weakly efficient. For the two dominated alternatives, *A* and *C*, *C* is better due to its higher efficiency value. Geometrically, the calculation of the efficiency value is based on the frontier facet with

Table 1
Data and various performance measures for the given example.

Alternative	Y_1	Y_2	DEA efficiency	Absolute distance		Relative distance	
				Ideal	Anti-ideal	Ideal	Anti-ideal
A	2	4	0.8 (5)	$\sqrt{2146}/130$ (5)	$\sqrt{2836}/130$ (5)	40/90 (5)	50/90 (5)
B	4	5	1 (1)	$\sqrt{225}/130$ (1)	$\sqrt{6625}/130$ (2)	11/90 (1)	79/90 (1)
C	4	4	0.92 (4)	$\sqrt{346}/130$ (2)	$\sqrt{5536}/130$ (4)	18/90 (3)	72/90 (3)
D	5	3	1 (1)	$\sqrt{484}/130$ (3)	$\sqrt{6714}/130$ (1)	14/90 (2)	76/90 (2)
E	5	2	$1-\varepsilon$ (3)	$\sqrt{1089}/130$ (4)	$\sqrt{6109}/130$ (3)	21/90 (4)	69/90 (4)

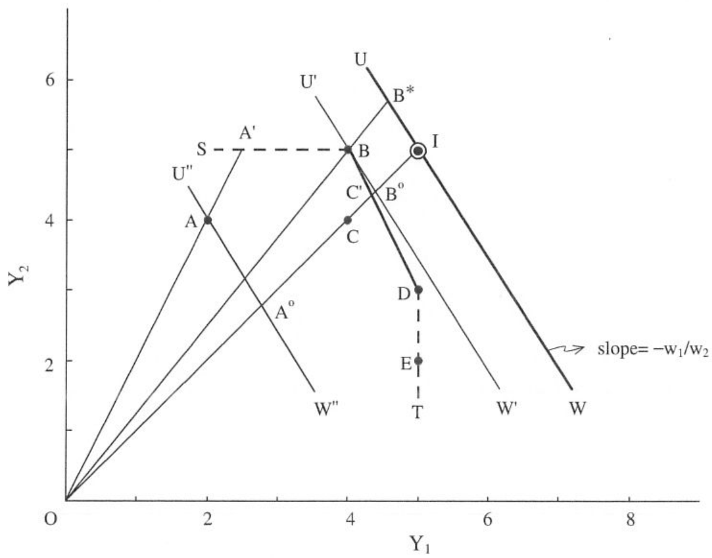

Fig. 1. Geometric interpretation of the relative performance compared with the ideal, *I*.

which the alternative is compared. Since the efficiency value is generally determined from different frontier facets, different weights are applied in calculating the efficiency. Hence, the alternatives should not be ranked simply by their efficiency values [19,20].

In the next section, we introduce a method for generating a common set of weights, based on which all alternatives are comparable.

3. Absolute distance

The first task in calculating the distance between an alternative and the ideal or the anti-ideal is to find the ideal and the anti-ideal. The anti-ideal, which has the smallest value in all criteria, is easier to determine. The origin is the theoretical anti-ideal because it has a value of zero for every criterion. The ideal, on the other hand, is difficult to identify because not every criterion has a theoretical ceiling.

Consider again a set of n alternatives with m criteria. The performance of alternative j in criterion i has a value of Y_{ij}. Let $Y_i^* = \max\{Y_{ij}, j = 1, \ldots, n\}$ denote the largest value that appears in the ith criterion. Then, $I = (Y_1^*, Y_2^*, \ldots, Y_m^*)$ is empirically the ideal alternative because it has the best performance in every criterion. For example, the ideal alternative generated from the five alternatives in Table 1 is $I = (5, 5)$. This derived alternative is the smallest dominating alternative, which will be dominated by any other dominating alternative. A comparison to this smallest dominating alternative will produce the highest performance value for each alternative. Therefore, it is a convincing ideal for measuring performance.

By the same token, if an empirical, rather than the theoretical, anti-ideal is preferred, then one can define $I^- = (Y_1^-, Y_2^-, \ldots, Y_m^-)$ as the anti-ideal, where $Y_i^- = \min\{Y_{ij}, j = 1, \ldots, n\}$ is the smallest value that appears in the ith criterion. This anti-ideal is the largest anti-ideal dominated by all observed alternatives; any other anti-ideal will be dominated by it.

One of the most popular approaches for ranking alternatives is compromise programming, which is based on the distance between the alternative and the ideal. Alternatives with a shorter distance to the ideal are considered better than those with a longer distance. There are several variations of this approach [10,11,21]. Referring to Eq. (1), a weight is usually applied to the values of each criterion to make all criteria comparable. When no prior information is available, one can use the information contained in the observations to generate the weight.

Consider the example in Table 1. Let w_1 and w_2 be the weights of criteria Y_1 and Y_2, respectively. The squared distance between alternative $A = (2, 4)$ and the ideal $I = (5, 5)$ is $[w_1(5-2)]^2 + [w_2(5-4)]^2 = 9w_1^2 + w_2^2$. Calculating the squared distance for the other four alternatives results in a total squared distance of $11w_1^2 + 15w_2^2$. For general cases, the total squared distance from all alternatives is $\sum_{j=1}^{n}[\sum_{i=1}^{m} w_i^2(Y_i^* - Y_{ij})^2]$. To exclude the trivial solution of $w_i^* = 0, i = 1, \ldots, m$, one can require the aggregate performance of the ideal alternative to have a value of 1; that is, $\sum_{i=1}^{m} w_i Y_i^* = 1$. Thus, rather than assigning the weights beforehand, they are obtained by minimizing the total squared distance to the ideal:

$$\begin{aligned} \text{min.} \quad & \sum_{j=1}^{n}\left[\sum_{i=1}^{m} w_i^2 (Y_i^* - Y_{ij})^2\right] \\ \text{s.t.} \quad & \sum_{i=1}^{m} w_i Y_i^* = 1 \\ & w_i \geqslant \varepsilon, \quad i = 1, \ldots, m \end{aligned} \tag{3}$$

The small quantity ε is introduced so that no criterion is ignored.

Model (3) is a quadratic program. Many computer programs can be applied to find a solution. After the optimal weights w_i^*, $i = 1, \ldots, m$, are solved, the distance between each alternative and the ideal can be calculated; the ranks of the alternatives can then be determined. Due to its special structure, the optimal solution of Model (3) can be derived from the Kuhn–Tucker conditions directly. Let λ be the Lagrangian multiplier. It is easy to derive:

$$w_i^* = \left[Y_i^* \Big/ 2\sum_{j=1}^{n}(Y_i^* - Y_{ij})^2\right]\lambda, \quad i = 1, \ldots, m \tag{4}$$

where $\lambda = 1/\sum_{i=1}^{m}[(Y_i^*)^2/2\sum_{j=1}^{n}(Y_i^* - Y_{ij})^2]$. Note that w_i^* will always be greater than zero, unless Y_i^* =0. Therefore, the lower bound constraint $w_i \geqslant \varepsilon$ is automatically satisfied. For Y_i^* =0, which means that all alternatives have a value of zero for the ith criterion, the criterion can be deleted without affecting the evaluation.

Using the data in Table 1 as an example, the associated mathematical program for finding the suitable weights is:

$$\begin{aligned} \text{min.} \quad & 11w_1^2 + 15w_2^2 \\ \text{s.t.} \quad & 5w_1 + 5w_2 = 1 \\ & w_1, w_2 \geqslant \varepsilon. \end{aligned} \tag{5}$$

The optimal weights are $w_1^* = 15/130$ and $w_2^* = 11/130$. The ratio of the two weights is $w_1/w_2 = 15/11$, indicating that the scale of the first criterion must be enlarged by a factor of 15/11 to make the two criteria comparable. Using this set of weights, the distance from each alternative to the ideal, $[\sum_{i=1}^{m} w_i^2(Y_i^* - Y_{ij})^2]^{1/2}$, is calculated as shown in the fifth column of Table 1. The corresponding ranks appear in parentheses.

Compared with the results of DEA efficiency, the nondominated alternative B is also ranked first with the absolute-distance approach. The other nondominated alternative D, however, is not the second best; it is ranked third. The weakly efficient alternative E has a rank of 4, instead of 3 as in the DEA-efficiency approach. The fourth ranked alternative, C, jumps up to a rank of 2. The fifth ranked alternative, A, remains the same. Three of the five alternatives have different ranks, which is due to every alternative in the DEA-efficiency approach possible using different sets of weights for comparison, while the absolute-distance approach requires all alternatives to use the same set of weights. According to Cooper and Tone [19] and Adler et al. [20], results from different sets of weights are not suitable for ranking.

In compromise programming, alternatives are ranked according to their distance to the ideal or to the anti-ideal. The alternative which is closest to the ideal need not be the same as that farthest away from the anti-ideal. In this example, the origin is the theoretical anti-ideal alternative because it has the smallest value in both criteria. Using the weights of $w_1^* = 15/130$ and $w_2^* = 11/130$ obtained from Model (3), the distances to the anti-ideal for the five alternatives are $\sqrt{2836}/130$, $\sqrt{6625}/130$, $\sqrt{5536}/130$, $\sqrt{6714}/130$, and $\sqrt{6109}/130$, respectively. The corresponding ranks are 5, 2, 4, 1, and 3, respectively, as shown in the sixth column of Table 1. Except alternative A, the worst one, all alternatives have ranks different from those obtained from the distance to the ideal. Interestingly, the rankings are the same as those of the DEA-efficiency approach. However, this is only a coincidence.

The reason for obtaining different rankings is simply that the absolute-distance approach only considers the distance to the ideal, disregarding the distance to the origin. If one can find a distance measure which takes both the ideal and anti-ideal into consideration, then consistent rankings may be obtained. In the next section, we introduce a measure which represents the relative position of an alternative from the origin to the ideal.

4. Relative distance

In the preceding section, it was illustrated that in calculating the aggregate performance of an alternative, one should look at the distance to the ideal relative to the total distance from the origin, passing through the alternative, to the ideal, rather than merely using the absolute distance to the ideal. Chen and Hwang [22] took this into account in devising a method named TOPSIS. However, their method does not guarantee the result from the criterion closest to the ideal and the result from that farthest away from the anti-ideal to be the same, as pointed out by Opricovic and Tzeng [21].

The aggregate performance of any alternative is worse than that of the ideal, no matter what weights are applied to individual criteria. Thus, we have $P_j = \sum_{i=1}^m w_i Y_{ij} / \sum_{i=1}^m w_i Y_i^* < 1$, where P_j is the aggregate performance of the jth alternative relative to the ideal. The ideal alternative has a relative performance value of 1: $P^* = \sum_{i=1}^m w_i Y_i^* / \sum_{i=1}^m w_i Y_i^* = 1$. If we let $\sum_{i=1}^m w_i Y_i^*$, which is the aggregate performance of the ideal alternative, be equal to 1 to standardize the weights w_i, then $\sum_{i=1}^m w_i Y_{ij}$ becomes the relative performance of the jth alternative. The difference between P_j and 1, denoted by s_j, is the relative distance between the jth alternative and the ideal in terms of the aggregate performance. It is also the complementary performance of this alternative. The problem is then transformed to finding the set of weights w_i, $i = 1, \ldots, m$, which produce the smallest total squared difference between the relative performance of the alternative and that of the ideal. The associated model is:

$$\begin{aligned} \text{min.} \quad & \sum_{j=1}^{n} s_j^2 \\ \text{s.t.} \quad & \sum_{i=1}^{m} w_i Y_{ij} + s_j = 1, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} w_i Y_i^* = 1 \\ & w_i \geqslant \varepsilon, \quad i = 1, \ldots, m. \end{aligned} \tag{6}$$

Note that the distance variable s_j is always positive because every alternative $\mathbf{Y}_j$ is dominated by the ideal $I = \mathbf{Y}^*$. After the optimal weights w_i^*, $i = 1, \ldots, m$, are obtained, the relative performance of the jth alternative is calculated as $\sum_{i=1}^m w_i^* Y_{ij}$. The relative distance to the ideal is $s_j^* = 1 - \sum_{i=1}^m w_i^* Y_{ij}$.

Comparing Model (6) with the conventional DEA model without inputs, i.e., Model (2), it is noted that the constraints of the two models are essentially the same, except that Model (6) has an extra constraint associated with the ideal alternative. In the context of DEA, the ideal alternative is also included to construct the production frontier. The difference between the two models is the objective function; Model (2) maximizes the aggregate performance of each specific alternative in each calculation, while Model (6) minimizes the total squared complementary performance, or the total squared relative distance to the ideal, of all alternatives in one calculation. The weights used by each alternative in calculating the aggregate performance can be different in Model (2); they are the same in Model (6). They are the general consensus of the alternatives being evaluated. The same set of weights provides a common base for comparing different alternatives.

Geometrically, the frontier constructed by Model (2) is a set of connected facets, while that constructed by Model (6) is a single-facet hyperplane, $\sum_{i=1}^m w_i Y_i = 1$, passing through the ideal. Note that here w_i's in $\sum_{i=1}^m w_i Y_i = 1$ are constants and Y_i's are coordinates. Model (6) can be considered as a common-weight DEA model [23]. The hyperplane $\sum_{i=1}^m w_i Y_i = 1 - s_j$, which passes through alternative j, is parallel to the frontier $\sum_{i=1}^m w_i Y_i = 1$, with a distance of s_j. Since s_j represents the relative position of alternative j from the origin to its projection on the frontier, it is a relative distance measure. Substituting s_j in the objective function of Model (6) by $(1 - \sum_{i=1}^m w_i Y_{ij})$, or $(\sum_{i=1}^m w_i Y_i^* - \sum_{i=1}^m w_i Y_{ij})$, from the constraints and omitting the first set of constraints, Model (6) can be simplified to:

$$\begin{aligned} \text{min.} \quad & \sum_{j=1}^{n} \left[\sum_{i=1}^{m} w_i (Y_i^* - Y_{ij}) \right]^2 \\ \text{s.t.} \quad & \sum_{i=1}^{m} w_i Y_i^* = 1 \\ & w_i \geqslant \varepsilon, \quad i = 1, \ldots, m. \end{aligned} \tag{7}$$

The relative distance to the ideal, $\sum_{i=1}^m w_i (Y_i^* - Y_{ij})$, is the basis for ranking.

The aggregate performance, $\sum_{i=1}^m w_i Y_{ij}$, represents the relative distance of alternative j to the origin. Larger values imply a location farther away from the anti-ideal. Since $\sum_{i=1}^m w_i Y_{ij}$ is the complement of s_j, as is clear from the first set of constraints in Model (6), the alternative with the smallest distance to the ideal, s_j, obviously has the largest distance to the anti-ideal, $\sum_{i=1}^m w_i Y_{ij}$. Hence, the relative distances of an alternative to the ideal and to the anti-ideal produce the same rankings.

The consistent rankings hold not only for the weights obtained from the proposed method, but also for any other set of weights which satisfies the condition of $\sum_{i=1}^m w_i Y_i^* = 1$. Let $\hat{w}_i$, $i=1, \ldots, m$, be any set of weights such that $\sum_{i=1}^m \hat{w}_i Y_i^* = 1$. For any pair of alternatives j and k, suppose that the former has a smaller relative distance to the ideal than does the latter, $s_j < s_k$. In other words, alternative j is ranked higher than alternative k. Then, we have $\sum_{i=1}^m \hat{w}_i Y_{ij} = 1 - s_j > 1 - s_k = \sum_{i=1}^m \hat{w}_i Y_{ik}$. That is, alternative j has a larger relative distance to the anti-ideal than does alternative k. The former is still ranked higher than the latter in terms of the distance to the anti-ideal.

For cases where the origin is not suitable to be the anti-ideal, and the empirical anti-ideal, $I^- = (Y_1^-, Y_2^-, \ldots, Y_m^-)$, is preferred, then P_j, the aggregate performance of the jth alternative relative to the ideal, can be adjusted by the aggregate performance of the anti-ideal as: $P_j = \sum_{i=1}^m w_i (Y_{ij} - Y_i^-) / \sum_{i=1}^m w_i (Y_i^* - Y_i^-)$. The geometric meaning is a translation of the origin to I^-. In this case, the adjusted performance of the ideal, $\sum_{i=1}^{m} w_i(Y_i^* - Y_i^-)$, is set to 1 to standardize the weight w_i. Furthermore, since the scale of Y_{ij} could be very large or very small, which would make the lower bound ε in $w_i \geqslant \varepsilon$ difficult to determine, a relative bound, $w_i(Y_i^* - Y_i^-) \geqslant b$, which requires the contribution of each criterion to the aggregate performance to be greater than a proportion b, is recommended. The corresponding model becomes:

$$\begin{aligned} \text{min.} \quad & \sum_{j=1}^{n} s_j^2 \\ \text{s.t.} \quad & \sum_{i=1}^{m} w_i(Y_{ij} - Y_i^-) + s_j = 1, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} w_i(Y_i^* - Y_i^-) = 1 \\ & w_i(Y_i^* - Y_i^-) \geqslant b, \quad i = 1, \ldots, m. \end{aligned} \tag{8}$$

It is easy to prove that, with this adjustment, the obtained weights produce the same rankings for the criterion of "closer to the ideal is better" and that of "farther away from the anti-ideal is better".

Following Model (7), the mathematical program for calculating the optimal weights for the five alternatives in Table 1 is:

$$\begin{aligned} \text{min.} \quad & (3w_1 + w_2)^2 + (w_1)^2 + (w_1 + w_2)^2 + (2w_2)^2 + (3w_2)^2 \\ \text{s.t.} \quad & 5w_1 + 5w_2 = 1 \\ & w_1, w_2 \geqslant \varepsilon. \end{aligned} \tag{9}$$

By solving the Kuhn-Tucker conditions, the optimal weights are obtained as $w_1^* = 11/90$ and $w_2^* = 7/90$. The frontier for calculating the aggregate performance of each alternative is a straight line, $(11/90)Y_1 + (7/90)Y_2 = 1$, passing through the ideal with a slope of $-w_1/w_2 = -11/7$. The relative distance to the ideal for alternative j, s_j, is the ratio of the distance between the alternative and its projection on the frontier to the distance between the origin and the projection point on the frontier. Its complement, $1 - s_j = \sum_{i=1}^{m} w_i Y_{ij}$, is the relative distance to the anti-ideal, which is also the relative performance value of this alternative. The last two columns of Table 1 show the relative distances of the five alternatives to the ideal and anti-ideal, respectively, with their ranks parenthesized. As expected, the rankings based on these two distance measures are exactly the same.

In Fig. 1, every alternative is compared with its projection on the frontier, UW, in calculating the aggregate performance. For example, the performance value of B is the ratio of OB to OB^*, where B^* is the projection of B on the frontier. BB^*/OB^* is the relative distance to the ideal and OB/OB^* is the relative distance to the anti-ideal. Let $U'W'$ be a straight line passing through B and parallel to the frontier UW. This line is represented by $(11/90)Y_1 + (7/90)Y_2 = 1 - s_B$. Suppose that the line connecting the origin and the ideal point intersects line $U'W'$ at B^0. It can be shown that $OB/OB^* = OB^0/OI$. Since the length of OI has been rescaled to 1, OB/OB^* is equal to the length of OB^0, or $w_1Y_{1B} + w_2Y_{2B}$, which is the aggregate performance of B. Thus, comparing an alternative with the ideal is equivalent to comparing it with its projection on the frontier. Similarly, one can draw a parallel line $U''W''$, $(11/90)Y_1 + (7/90)Y_2 = 1 - s_A$, for alternative A, which intersects line OI at A^0. The length of OA^0 is the aggregate performance of alternative A, with a value of $w_1Y_{1A} + w_2Y_{2A}$. For the general case of n alternatives, n parallel hyperplanes are constructed; each has a distance of s_j to the frontier. The alternative with the shortest distance to the frontier has the highest rank.

For a set of weights $\hat{w}_i$, $i = 1, \ldots, m$, "$\sum_{i=1}^{m} \hat{w}_i Y_i^* = 1$" represents a hyperplane passing through the ideal I. Following the above discussion, the relative performance of an alternative is equal to the ratio of the distance between the origin and the alternative to that between the origin and the projection of the alternative on the hyperplane. This value is equal to $1 - s_j$, the complement of the distance between the parallel hyperplanes passing through the alternative and the ideal. Therefore, this set of weights produces the same rankings for the criteria of "closer to the ideal is better" and "farther away from the anti-ideal is better".

Another point which can be inferred from Fig. 1 is that criteria with larger variances obtain larger weights in the proposed method. Consider an extreme case that all alternatives are almost vertically scattered along the line of $Y_1 = 5$. In other words, all alternatives have similar values in Y_1 but different values in Y_2. Then, the frontier constructed from these alternatives will be an almost vertical line to the right of all alternatives with a slope approaching negative infinity. In this case, the weight of Y_1, w_1, approaches zero and the weight of Y_2, w_2, is a relatively large number. Thus, the contribution of Y_1 to the aggregate performance is negligible. This property coincides with a concept in MCDA that states that criteria with similar values for all alternatives are judged less important as they do not help in making a decision [10].

5. A comparison with TOPSIS

The method proposed in this paper uses an idea similar to that used in TOPSIS (technique for order preference by similarity to an ideal solution) [11]. This method has several variations [24,25]; however, the basic idea is the same. The procedure can be categorized into five steps.

(1) *Data standardization:* $r_{ij} = Y_{ij}/\sqrt{\sum_{j=1}^{n} Y_{ij}^2}$, $i = 1,\ldots, m$, $j = 1,\ldots, n$.
(2) *Data weighting:* $v_{ij} = w_i r_{ij}$, where $\sum_{i=1}^{m} w_i = 1$.
(3) *Ideal and anti-ideal determination:* $I = (v_1^*,\ldots,v_m^*)$ and $I^- = (v_1^-,\ldots,v_m^-)$, where $v_i^* = \max\{v_{ij}, j = 1,\ldots,n\}$ and $v_i^- = \min\{v_{ij}, j = 1,\ldots,n\}$ for desirable criteria and $v_i^* = \min\{v_{ij}, j = 1,\ldots,n\}$ and $v_i^- = \max\{v_{ij}, j = 1,\ldots,n\}$ for undesirable criteria.
(4) *Distance calculation:* $s_j^* = \sqrt{\sum_{i=1}^{m}(v_i^* - v_{ij})^2}$ and $s_j^- = \sqrt{\sum_{i=1}^{m}(v_{ij} - v_i^-)^2}$, $j = 1,\ldots,n$.
(5) *Ranking:* $c_j^* = s_j^-/(s_j^* + s_j^-)$, $j = 1,\ldots,n$.

There are several differences between TOPSIS and the method proposed in this paper.

First, the data in TOPSIS is standardized to eliminate the difference in scale of each criterion, while in the proposed method the scales are adjusted automatically by the weights associated with the criteria. Second, the weights in TOPSIS are specified beforehand and sum to one, while in the proposed method they are determined by the data and are not required to sum to one. Finally, and most importantly, the sum of the distances of each alternative to the ideal, s_j^*, and to the anti-ideal, s_j^-, is not the same for all alternatives for TOPSIS. Therefore, the rankings based on s_j^*, s_j^-, or c_j^*, the relative position of an alternative between the ideal and anti-ideal, may not be the same. In contrast, the distances to the ideal and anti-ideal for each alternative in the proposed method always sum to one. Consequently, the rankings based on the two distance measures are the same.

Consider an example that appeared in Jacquet-Lagrèze and Siskos [8]. Ten cars are to be ranked by six criteria: maximum speed (km), horse power (cv), space (m^2), gas consumption in town (lt/100 km), gas consumption at 120 km/h (lt/100 km), and price (francs), where the first three are desirable criteria and the last three are undesirable ones. Table 2 shows the data. By applying Model (8), the distance to the anti-ideal, $\sum_{i=1}^{m} w_i(Y_{ij} - Y_i^-)$, and the distance to the ideal, s_j, are calculated for each alternative. The results are shown in the second and third columns of Table 3, where numbers in parentheses are the corresponding ranks. As expected, the rankings from the two distance measures are the same. Note that the ideal and anti-ideal for an undesirable criterion are the minimal and maximal observations, respectively, in all alternatives. Hence, $(Y_{ij} - Y_i^-)$ and $(Y_i^* - Y_i^-)$ for the last three criteria in Model (8) must be replaced by absolute values to maintain the correct relationship. Let b in Model (8) be 0.01; the weights obtained for w_i, $i = 1,\ldots,6$, are 0.009763, 0.001, 0.002976, 0.00137, 0.001613, and 0.006393, respectively. Since the scales of the criteria are different, these weights do not necessarily represent the importance of the criteria; they must be adjusted.

The contribution of each criterion to the aggregate performance of the ideal is $w_i(Y_i^* - Y_i^-)$, which sum to one for all criteria. Therefore, $w_i(Y_i^* - Y_i^-)$ can be considered as the importance of criterion i. The values are 0.6346, 0.01, 0.01, 0.01, 0.01,

Table 2
Data for ten cars with six criteria [8].

No.	Maximum speed (km)	Horse power (CV)	Space (m^2)	Gas consumption in town (lt/100 km)	Gas consumption at 120 km/h (lt/100 km)	Price (1000 francs)
1	173	10	7.88	11.4	10.01	49.5
2	176	11	7.96	12.3	10.48	46.7
3	142	5	5.65	8.2	7.30	32.1
4	148	7	6.15	10.5	9.61	39.15
5	178	13	8.06	14.5	11.05	64.7
6	180	13	8.47	13.6	10.40	75.7
7	182	11	7.81	12.7	12.26	68.593
8	145	11	8.38	14.3	12.95	55.0
9	161	7	5.11	8.6	8.42	35.2
10	117	3	5.81	7.2	6.75	24.8

Table 3
Rankings for the car example from the proposed method and TOPSIS.

No.	Proposed method		TOPSIS		
	s_j	$1 - s_j$	s_j^*	s_j^-	c_j^*
1	0.2615 (2)	0.7385 (2)	0.0505 (3)	0.0870 (5)	0.6329 (4)
2	0.2151 (1)	0.7849 (1)	0.0443 (2)	0.0934 (3)	0.6783 (2)
3	0.4558 (8)	0.5442 (8)	0.0519 (5)	0.0922 (4)	0.6401 (3)
4	0.4457 (7)	0.5543 (7)	0.0511 (4)	0.0824 (6)	0.6173 (5)
5	0.3123 (5)	0.6877 (5)	0.0797 (7)	0.0790 (8)	0.4979 (7)
6	0.3596 (6)	0.6404 (6)	0.1014 (10)	0.0784 (9)	0.4359 (9)
7	0.3004 (4)	0.6996 (4)	0.0873 (9)	0.0821 (7)	0.4847 (8)
8	0.5763 (9)	0.4237 (9)	0.0758 (6)	0.0541 (10)	0.4164 (10)
9	0.2921 (3)	0.7079 (3)	0.0334 (1)	0.0975 (2)	0.7447 (1)
10	0.6525 (10)	0.3475 (10)	0.0808 (8)	0.1014 (1)	0.5564 (6)

and 0.3254 for the six criteria, respectively. These numbers indicate that the first criterion, maximum speed, is the most important, that the last criterion, price, is the second most important, and that the others are equally unimportant for generating the most favorable aggregate performance for all alternatives in a compromise sense. The standard deviations of the six criteria are 21.54, 3.41, 1.29, 2.62, 1.99, and 16.81, respectively. The first criterion has the largest value, followed by the last. All the others have significantly smaller values. This explains why the first criterion generates the largest weight, the last criterion generates the second largest weight, and all other criteria generate very small weights from Model (8).

In applying TOPSIS to rank the ten alternatives, the weights 0.6346, 0.01, 0.01, 0.01, 0.01, and 0.3254 are used in the second step, *data weighting*. The results of ranking are shown in the last three columns of Table 3. Clearly, the rankings from the criterion of "closer to the ideal is better" are different from those of "farther away from the anti-ideal is better". The largest difference occurs at alternative 10, where the former ranked it eighth while the latter ranked it first. The final ranking, as shown in the last column, are a compromise of these two types of rankings. The best is alternative 9, which was ranked first by the former criterion and second by the latter criterion.

The final rankings from TOPSIS are also different from those of the proposed method. The proposed method ranked alternative 2 the best while TOPSIS ranked it second. In contrast, TOPSIS ranked alternative 9 the best while the proposed method ranked it third. From the viewpoint of identifying the best alternative, the difference between these two methods, in this example, is not much.

Different methods usually lead to different results. It is inappropriate to say which method is better because every method has a different underlying theory or assertion. However, some methods are more suitable than others for certain cases. The proposed method uses observations to generate weights; no pre-specified weights are required. Furthermore, the proposed method produces the same rankings for the criteria of "closer to the ideal is better" and "farther away from the anti-ideal is better". From this point of view, the proposed method is suitable for, at least, two cases. First, when the weights of the criteria are difficult to determine due to either insufficient prior information or contradictory expert opinions. Second, when the decision maker is not sure whether an alternative with maximum aggregate performance, i.e., farther away from the anti-ideal, or with minimum regret, i.e., closer to the ideal, is better.

Since the weights of the proposed method are generated from the data, undesirable data may produce undesirable results. For example, suppose that most factories, except one or two, are producing a lot of pollution. The weight generated for the criterion pollution will be relatively small, giving most factories a favorable evaluation. This is definitely not correct. Therefore, whenever *a priori* weights are available, the weights should not be generated from the data. Instead, the *a priori* weights should be used. As discussed in Section 4, the *a priori* weights $\hat{w}_i, i = 1, \ldots, m$, can still be used in the proposed method to produce consistent rankings.

6. Conclusion

How to compare various alternatives in MCDA is always controversial due to the incomparability of criteria. There are two types of weight acquired for representing the importance of each criterion: *a priori* weights determined by experts and *a posteriori* weights obtained from the data. This paper adopted the *a posteriori* approach. The general idea is to find a set of weights which produces the aggregate performance for all alternatives to be as close to the ideal as possible. The problem was formulated as a compromise program. After the weights are obtained, the aggregate performance of each alternative is calculated for ranking.

The major finding of this paper is that the conventional idea of seeking the shortest absolute distance between the alternative and the ideal may produce results which are different from those obtained by seeking the longest absolute distance between the alternative and the anti-ideal. When the measure of distance is changed from absolute to relative, that is, the relative position of the alternative between the anti-ideal and the ideal, then the resultant rankings from the two ideas are consistent. It was also shown that the relative distance produces consistent rankings for any set of weights, no matter how they are determined. Although the TOPSIS method also calculates the relative distance of each alternative between the ideal and anti-ideal, the results from the criteria "closer to the ideal is better" and "farther away from the anti-ideal is better" are not necessarily the same because the sum of the distances of an alternative to the ideal and to the anti-ideal is not constant for all alternatives.

The proposed method has several other advantages. One is that criteria with similar values for all alternatives generate smaller weights. This is in accord with a concept in MCDA that states that criterion with similar values for all alternatives are less helpful in making a decision. Another advantage which is common to all *a posteriori* weight approaches is that pre-determined weights are not required. Hence, it is suitable for cases where no prior information can be used for determining the weights.

Finally, since the weights used for calculating the aggregate performance for the alternatives are the most favorable for all alternatives in a compromise sense, the resultant rankings are convincing. Moreover, the weights are not subjectively determined by humans, which sometimes creates controversy; hence, the results are more acceptable when the alternatives are people or organizations.

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